From the 2017 Administration

AP® CollegeBoard

Calculus AB

Practice Exam

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Class: Grade 11 Calculus
班级：微积分 11 年级

11?	13	17?	22	23	24	25?	27?	28?	29	30	77?	81
84	85?	86?	87	88?								

CALCULUS AB
SECTION I, Part A
Time—1 hour
Number of questions—30
NO CALCULATOR IS ALLOWED FOR THIS PART OF THE EXAM.

1. If $f(x) = (2x^2+5)^7$, then $f'(x) =$

$f'(x) = 7(2x^2+5)^6$

答案 (B)

2. $\int \frac{1}{3x+12} dx =$

$$\int \frac{1}{3x+12} dx = \frac{1}{3}\int \frac{d(x+4)}{x+4} dx = \frac{1}{3}\ln|x+4| + C$$

答案 (B)

3. If $f(x) = \frac{5-x}{x^3+2}$ then $f'(x) =$

$$f'(x) = \frac{-(x^3+2)-(5-x)2x}{(x^3+2)^2} = \frac{2x^3-15x^2-2}{(x^3+2)^2}$$

答案 (C)

t	0	0.5	2	3
v(t)	20	60	40	30

4. The table above gives the velocity v(t), in miles per hour, of a truck at selected times t, in hours. Using a trapezoidal sum with the three subintervals indicated by the table, what is the approximate distance, in miles, the truck traveled from time t = 0 to t = 3?

t	0	0.5	2	3
v(t)	20	60	40	30
s(t)		20	75	35
sum				130

答案 (B)

5. If $f(x) = \sin(x^2+\pi)$, then $f'\left(\sqrt{2\pi}\right) =$

$f(x) = \sin(x^2+\pi) = -\sin(x^2)$

$f'(x) = -\cos x^2\,(2x)$

$f'(2\pi) = -\{\cos[2\pi]\}\left(2\sqrt{2\pi}\right) = -2\sqrt{2\pi}$

答案 (A)

6. If *f* is the function given by $f(x) = 3x^2 - x^3$, then the average rate of change of *f* on the closed interval[1 , 5]is

$f(1) = 3-1 = 2$

$f(5) = 75-125 = -50$

$$\frac{[f(5)-f(1)]}{4} = \frac{-50-2}{4} = -13$$

答案 (B)

7. If $\int_4^{-10} g(x)dx = -3$ and $\int_4^6 g(x)dx = 5$, then $\int_{-10}^6 g(x)dx =$

$$\int_{-10}^{6} g(x)dx = \int_{-10}^{4} g(x)dx + \int_{4}^{6} g(x)dx = 3+5 = 8$$

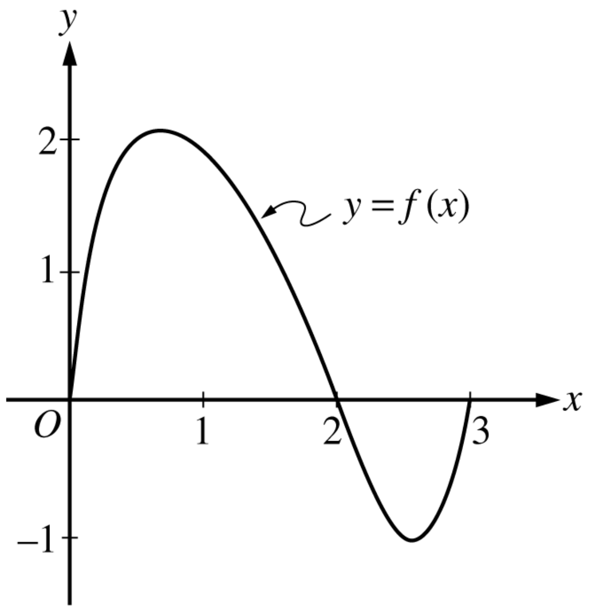
答案 (D)

8. If f is the function given by $f(x) = e^{x/3}$, which of the following is an equation of the line tangent to the graph of f at the point(3 ln 4,4) ?

$$f'(x) = e^{\frac{x}{3}}\left(\frac{1}{3}\right)$$

$$f'(4) = e^{\frac{3\ln 4}{3}}\left(\frac{1}{3}\right) = \frac{4}{3}$$

答案 (A)



9. The graph of a function f is shown above. Which of the following expresses the relationship between

$$\int_0^2 f(x)dx\,, \int_0^3 f(x)dx\,, and \int_2^3 f(x)dx$$

答案 (D)

10. $\int_0^2 (x^3+1)^{1/2} x^2 dx$

$$\int_0^2 (x^3+1)^{1/2} x^2 dx = \frac{1}{3}\int_0^2 (x^3+1)^{1/2} d(x^3+1) = \left(\frac{1}{3}\frac{(x^3+1)^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^2$$

$$= \left[\frac{2}{9}(x^3+1)^{\frac{3}{2}}\right]_0^2 = \frac{52}{9}$$

答案 (A)

11. If $x^2 + xy - 3y = 3$, then at point (2,1), $\frac{dy}{dx} =$

$$\frac{dy}{dx} = 2x + y + x\frac{dy}{dx} - 3\frac{dy}{dx} = 0$$

$$(x-3)\frac{dy}{dx} = -2x - y$$

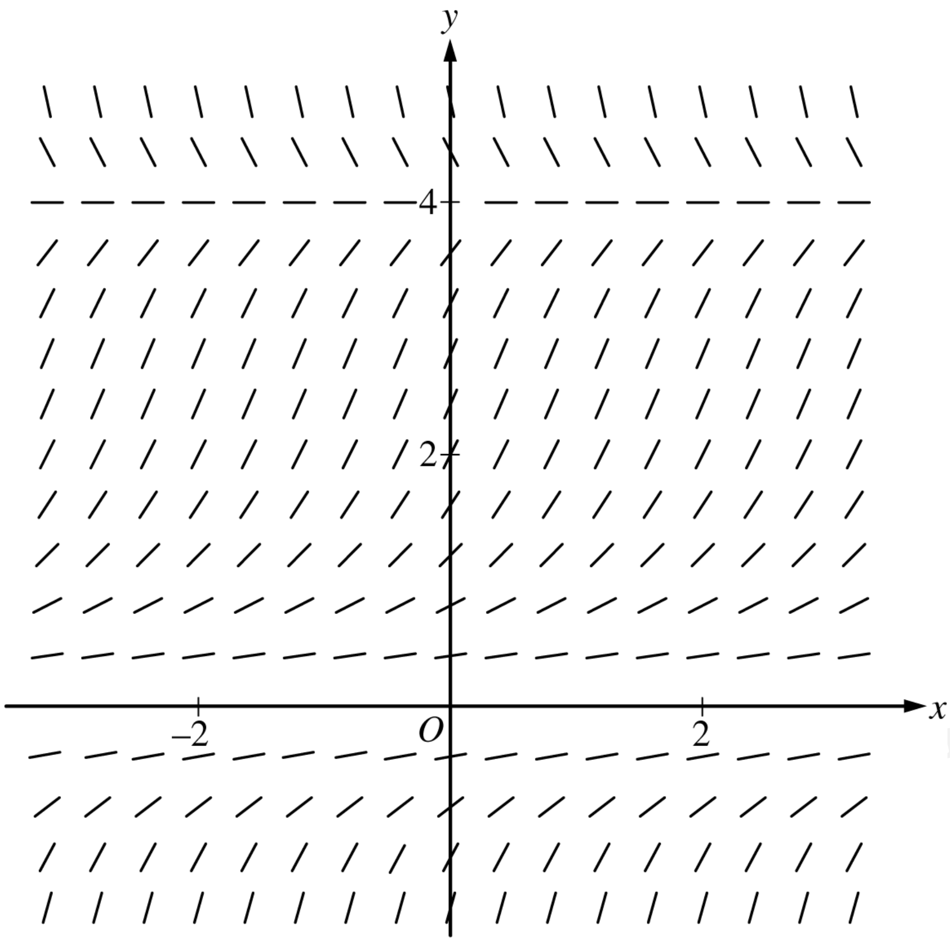
$$\frac{dy}{dx} = \frac{-2x-y}{x-3}$$

$$\left(\frac{dy}{dx}\right)_{(2,1)} = \frac{-4-1}{2-3} = 5$$

答案 (A)

12. The number of gallons of water in a storage tank at time t, in minutes, is modeled by $w(t) = 25 - t^2$ for $0 \le t \le 5$. At what rate, in gallons per minute, is the amount of water in the tank changing at time $t = 3$ minutes?

$w'(t) = -2t$
$w'(3) = -6$
答案 (D)



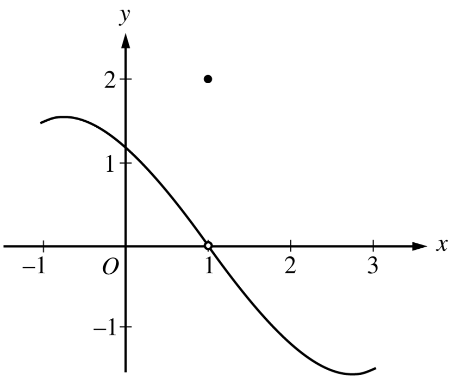
13. Shown above is a slope field for which of the following differential equations?

$$\frac{dy}{dx} = \frac{y^2(4-y)}{4}$$

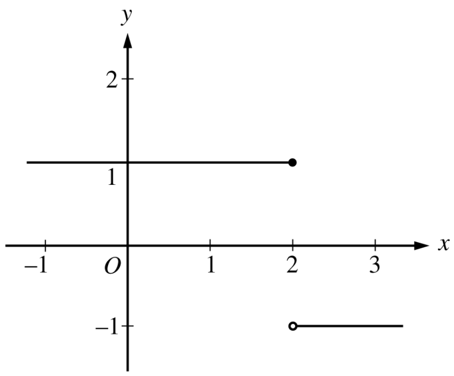
答案 (D)

14. The weight of a population of yeast is given by a differentiable function y, where y (t) is measured in grams and t is measured in days. The weight of the yeast population increases according to the equation $\frac{dy}{dx} = ky$, where k is a constant. At time $t = 0$, the weight of the yeast population is 120 grams and is increasing at the rate of 24 grams per day. Which of the following is an expression for y (t)?

$120\ e^{t/5} =$
答案 (B)



Graph of f



Graph of g

15. The graphs of the functions f and g are shown in the figures above. Which of the following statements is false??

$\lim_{x\to 1}(f(x)g(x+1))$ does not exist

答案 (C)

16. Let *f* be the function defined by $f(x) = -3 + 6x^2 - 2x^3$. What is the largest open interval on which the graph of *f* is both concave up and increasing?

$f'(x) = 12x - 6x^2$

$f''(x) = 12 - 12x$

(0,1)

答案 (A)

17. A particle moves along the x-axis so that at time t > 0 t its position is given by $x(t) = 12e^{-t}\sin t$. What is the first time t at which the velocity of the particle is zero?

$x'(t) = -12e^{-t}\sin t + 12e^{-t}\cos t = 0$

$\sin t = \cos t \Rightarrow t = \pi/4$

答案 (A)

18. Let F be the function given by $F(x) = \int_0^x (\tan(5t)\sec(5t) - 1)dt$. Which of the following is an expression for $F'(x)$?

$\tan(5x)\sec(5x) - 1$

答案 (D)

19. Let *f* be the function given by $f(x) = 2\cos x + 1$. What is the approximation for *f*(1.5) found by using the line tangent to the graph of *f* at $x = \pi/2$?

$f'(x) = -2\sin x$

$f'(\pi/2) = -2\sin\left(\frac{\pi}{2}\right) = -2$

$\pi - 2$

答案 (C)

20. Let *f* be the function given by $f(x) = \frac{x-2}{2|x-2|}$. Which of the following is true?

f has a jump discontinuity at $x = 2$.

答案 (C)

21. If $f(x) = \ln x$. Then $\lim_{x\to 3}\frac{f(x)-f(3)}{x-3}$ is

$$\lim_{x\to 3}\frac{f(x)-f(3)}{x-3} = \frac{d(f)}{dx}\bigg|_{x=3} = \frac{1}{|x|}\bigg|_{x=3} = \frac{1}{3}$$

答案 (A)

22. Which of the following is the solution to the differential equation $\frac{dy}{dx}=\frac{2y}{2x+1}$ with the initial condition $y(0)=e$ for $x>-\frac{1}{2}$?

$$\frac{dy}{dx}=\frac{2y}{2x+1} \Rightarrow \frac{dy}{2y}=\frac{dx}{2x+1} \Rightarrow \frac{d(2y)}{2y}=\frac{d(2x+1)}{2x+1}$$

$$\Rightarrow \ln 2y=\ln(2x+1)+C_1 \Rightarrow 2y=C_1|2x+1| \xRightarrow{y(0)=e} 2e=C_1|2\times 0+1|$$

$$\Rightarrow 2e=C_1$$

$$\Rightarrow 2y=2e|2x+1| \xRightarrow{x>-\frac{1}{2}} 2y=2e(2x+1) \Rightarrow y=e(2x+1)$$

答案 (B)

23. The region enclosed by the graphs of $y=x^2$ and $y=2x$ is the base of a solid. For the solid, each cross section perpendicular to the y-axis is a rectangle whose height is 3 times its base in the x - plane. Which of the following expressions gives the volume of the solid?

$$\begin{cases} y=x^2 \\ y=2x \end{cases} \Rightarrow \begin{cases} (0,0) \\ (2,4) \end{cases}$$

The region enclosed by the graphs of $y=x^2$ and $y=2x$ is:

$$\int_0^2 (2x-x^2)\,dx=\int_0^4 \left(\frac{y}{2}-\sqrt{y}\right)dy$$

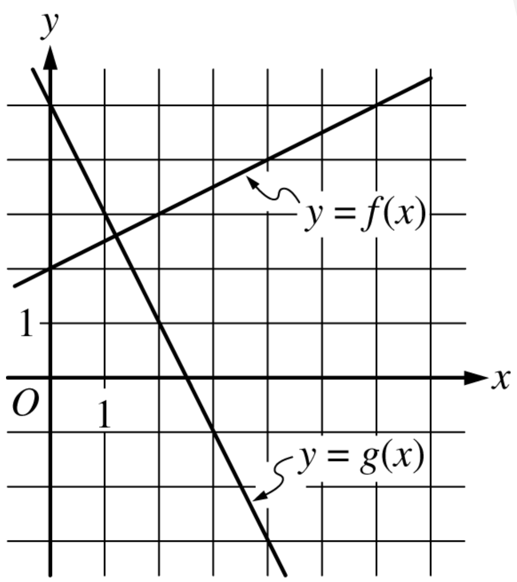
答案 (A)

24. If the average value of a continuous function f on the interval $[-2, 4]$ is 12, what is $\int_{-2}^{4}\frac{f(x)}{8}dx$?

$$\int_{-2}^{4} f(x)dx=12\times\big(4-(-2)\big)=72$$

72/8=9

答案 (C)



25. The figure above shows the graphs of the functions f and g. If $h(x)=f(x)g(x)$, then $h'(2)=$

$$f(x)=\frac{1}{2}(x+2)$$

$$g(x)=-2(x+5)$$

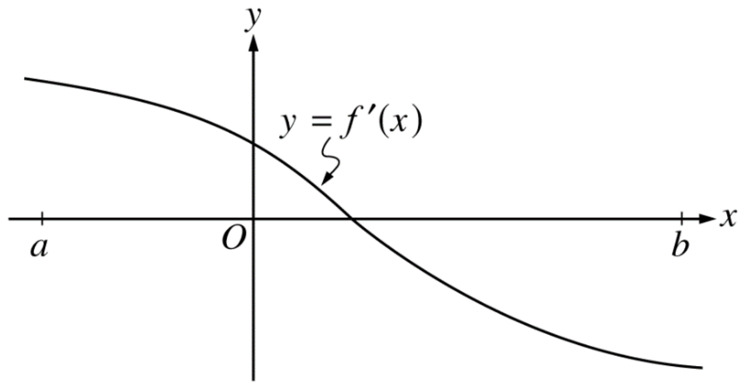
$$h(x)=f(x)g(x)=\frac{1}{2}(x+2)[-2(x+5)]=-(x+2)(x+5)$$

$$=-x^2-7x-10$$

$h'(x) = -2x - 7$
$h'(2) = -4 - 7 = -11$
答案 (D)

26. $\lim_{x\to\infty} \frac{\ln(e^{3x}+x)}{x}$

$\lim_{x\to\infty} \frac{\ln(e^{3x})}{x} = 3$
答案 (C)



27. The graph of f', the derivative of the function *f*,is shown in the figure above. Which of the following statements must be true?
I. *f* is continuous on the open interval(*a* ,).
II. II.*f* is decreasing on the open interval(*a* , *b*)
III. III.The graph of *f* is concave down on the open interval(*a* , *b*)

I and II only
答案 (B)

28. An isosceles right triangle with legs of length *s* has area $A = \frac{1}{2}s^2$. At the instant when $s = \sqrt{32}$ centimeters, the area of the triangle is increasing at a rate of 12 square centimeters per second. At what rate is the length of the hypotenuse of the triangle increasing, in centimeters per second, at that instant?

At the instant when $s = \sqrt{32}$, $A = \frac{1}{2}s^2 = 16$

$$\frac{dA}{dt} = 12 = \frac{d\left(\frac{1}{2}s^2\right)}{dt} = \frac{1}{2}(2s)\frac{ds}{dt} \Rightarrow \frac{ds}{dt} = \frac{12}{s} = \frac{12}{\sqrt{32}} = \frac{3}{\sqrt{2}}$$

答案 (B)

29. The graph of which of the following functions has exactly one horizontal asymptote and no vertical asymptotes?

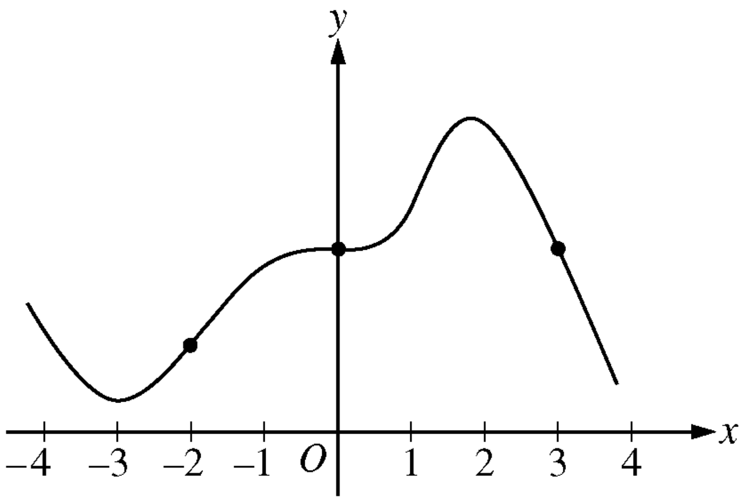
$y = \frac{1}{x^2+1} = (x^2+1)^{-1}$
$y' = -1(x^2+1)^{-2} = \frac{-1}{(x^2+1)^2}$
y' is continuous so no vertical asymptotes.
$y' = 0 \Rightarrow x = \pm\infty$
$\lim_{x\to\pm\infty} \frac{1}{x^2+1} = 0$: one horizontal asymptotes of $y = 0$
答案 (A)

30. For a certain continuous function *f*, the right Riemann sum approximation of

$\int_0^2 f(x)dx$ with n subintervals of equal length is $\frac{2(n+1)(3n+2)}{n^2}$ for all n. What is the value of $\int_0^2 f(x)dx$?
$\lim_{n\to\infty}\frac{2(n+1)(3n+2)}{n^2}=\lim_{n\to\infty}\frac{6n^2+10n+4}{n^2}=6$ 答案 (B)

END OF PART A
IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.
DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB
SECTION I, Part B
Time—45 minutes
Number of questions—15
A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.



Graph of f

76. The graph of a differentiable function *f* is shown in the figure above. Which of the following is true?

$f'(3) < f'(0) < f'(-2)$
答案 (D)

77. Let *H* (x) be an antiderivative of $\frac{x^3+\sin x}{x^2+2}$. If H(5)=π, then H(2)

-5.867
答案 (B)

78. The continuous function f is positive and has domain x >0. If the asymptotes of the graph of f are x=0 and y=2, which of the following statements must be true?

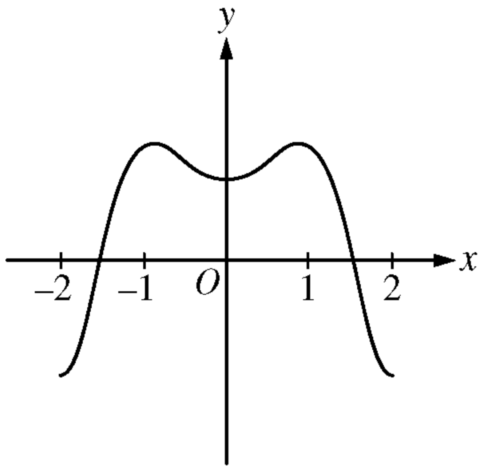
Asymptote at x=0: $\lim_{x\to 0^+} f(x) = \infty$
Asymptote at y=2: $\lim_{x\to\infty} f(x) = 2$
答案 (C)

79. A file is downloaded to a computer at a rate modeled by the differentiable function *f(t)*, where *t* is the time in seconds since the start of the download and *f(t)* is measured in megabits per second. Which of the following is the best interpretation of $f'(5) = 2.8$?

At time *t* = 5 seconds, the rate at which the file is downloaded to the computer is increasing at a rate of 2.8 megabits per second per second.
答案 (B)

80. The function f has first derivative given by $f'(x) = x^4 - 6x^2 - 8x - 3$. On what intervals is the graph of f concave up?

$f''(x) = 4x^3 - 12x - 8 > 0$
$\Rightarrow 4(x+1)(x-2)(x+1) > 0$
$\Rightarrow (2,\infty)$ only
答案 (A)



Graph of f

81. The graph of the function f is shown above for $-2 \le x \le 2$. Which of the following could be the graph of an antiderivative of f?

从-2到-1的积分应该是0，所以感觉只有C有点像?
答案（？？）

82. A particle travels along a straight line with velocity $v(t) = 3e^{-t/2}\sin(2t)$ meters per second. What is the total distance, in meters, traveled by the particle during the time interval $0 \le t \le 2$ seconds?

Integration by parts:

$$\frac{d}{dx}[uv] = u\frac{dv}{dx} + v\frac{du}{dx} = uv' + vu'$$

$$\Rightarrow uv = \int uv'dx + \int vu'dx = \int udv + \int vdu$$

$$\int udv = uv - \int vdu$$

- Step 1

$u = \sin(2t)\,,du = 2\cos 2t\,dt$

$v = e^{-\frac{t}{2}}, dv = e^{-\frac{t}{2}}\left(\frac{-1}{2}\right)dt$

$$s(t) = \int_0^2 3e^{-\frac{t}{2}}\sin(2t)\,dt = 3(-2)\int_0^2 \left[e^{-\frac{t}{2}}\left(\frac{-1}{2}\right)\right]\sin(2t)\,dt$$

$$= -6\int_0^2 \sin(2t)\,d(e^{-\frac{t}{2}})$$

$$= -6\left[(\sin 2t)\left(e^{-\frac{t}{2}}\right) - \int_0^2 e^{-\frac{t}{2}}d(\sin 2t)\right]$$

$$= -6\left[(\sin 2t)\left(e^{-\frac{t}{2}}\right) - 2\int_0^2 e^{-\frac{t}{2}}\cos 2t\,dt\right] \qquad (1)$$

- Step 2

$u = \cos(2t)$, $du = -2\sin 2t\, dt$

$v = e^{-\frac{t}{2}}$, $dv = e^{-\frac{t}{2}}\left(\frac{-1}{2}\right)dt$

$$\int_0^2 e^{-\frac{t}{2}}\cos 2t\, dt = -2\int_0^2 e^{-\frac{t}{2}}\left(\frac{-1}{2}\right)\cos 2t\, dt = -2\int_0^2 \cos 2t\, d(e^{-\frac{t}{2}})$$

$$= -2\left[(\cos 2t)e^{-\frac{t}{2}} - \int_0^2 e^{-\frac{t}{2}} d(\cos 2t)\right]$$

$$= -2\left[(\cos 2t)e^{-\frac{t}{2}} + 2\int_0^2 e^{-\frac{t}{2}}\sin 2t\, dt\right]$$

$$= -2\left[(\cos 2t)e^{-\frac{t}{2}} + \frac{2}{3}\int_0^2 3e^{-\frac{t}{2}}\sin 2t\, dt\right]$$

$$= -2\left[(\cos 2t)e^{-\frac{t}{2}} + \frac{2}{3}s(t)\right] \quad (2)$$

- Step 3

$$(1) \Rightarrow s(t) = -6\left[(\sin 2t)\left(e^{-\frac{t}{2}}\right) - 2\int_0^2 e^{-\frac{t}{2}}\cos 2t\, dt\right]$$

$plug\ (2)\ into\ (1)\ \Rightarrow$

$$s(t) = -6\left[(\sin 2t)\left(e^{-\frac{t}{2}}\right) - 2\left\{-2\left[(\cos 2t)e^{-\frac{t}{2}} + \frac{2}{3}s(t)\right]\right\}\right]$$

$$s(t) = -6\left[(\sin 2t)\left(e^{-\frac{t}{2}}\right) + 4(\cos 2t)e^{-\frac{t}{2}} + \frac{8}{3}s(t)\right]$$

$$s(t) = -6(\sin 2t)\left(e^{-\frac{t}{2}}\right) - 24(\cos 2t)e^{-\frac{t}{2}} - 16s(t)$$

$$17s(t) = -6(\sin 2t)\left(e^{-\frac{t}{2}}\right) - 24(\cos 2t)e^{-\frac{t}{2}}$$

$$s(t) = \left[-\frac{6}{17}\sin 2t\left(e^{-\frac{t}{2}}\right) - \frac{24}{17}(\cos 2t)e^{-\frac{t}{2}}\right]_0^2$$

$$s(t) = \left(\frac{6}{17}\sin 0\,(e^0) + \frac{24}{17}(\cos 0)e^0\right) - \left(\frac{6}{17}\frac{\sin 4}{e} + \frac{24}{17}\frac{\cos 4}{e}\right)$$

$$s(t) = \frac{24}{17} - \left(\frac{6}{17}\frac{\sin 4}{e} + \frac{24}{17}\frac{\cos 4}{e}\right) \approx 2.261$$

答案 (D)

83. Let f be a function with derivative given by $f'(x) = \frac{x^3-8x^2+3}{\sqrt{x^3+1}}$ for -1<x<9. At what value of x does f attain a relative maximum?

$$f'(x) = \frac{x^3-8x^2+3}{\sqrt{x^3+1}} = 0 \Rightarrow \begin{cases} x^3 - 8x^2 + 3 = 0 \\ x \neq -1 \end{cases}.$$

$x \approx -0.591, 0.638, 7.953$

for -1<x<9 $x \approx 7.953$

答案 (D)

84. The number of bacteria in a container increases at the rate of R (t) bacteria per hour. If there are 1000 bacteria at time $t = 0$, which of the following expressions gives the number of bacteria in the container at time $t = 3$ hours?

$$1000 + \int_0^3 R(t)dt$$

答案 (D)

85. The function g is continuous on the closed interval [1, 4] with $g(1)=5$ and $g(4)=8$. Of the following conditions, which would guarantee that there is a number c in the open interval (1,4) where $g'(c)=1$?

没明白，如果在[1,4]区间上是一条直线呢？这个时候$g'(c)=4/3$?

答案 B (g is differentiable on the open interval (1 , 4).)

$f''(x) = x(x-1)^2\,(x+2)^3$
$g''(x) = x(x-1)^2(x+2)^3 + 1$
$h''(x) = x(x-1)^2(x+2)^3 - 1$

86. The twice-differentiable functions f, g, and h have second derivatives given above. Which of the functions f, g, and h have a graph with exactly two points of inflection?

先画出f''的图像，有2个拐点。
f''向下平移就变为h''，就多于两个拐点啦。
再判断g''是OK的
f and g only
答案（C）

x	1	2	3	4	5
$f(x)$	9	4	0	-3	-5

87. The table above gives values of a function f at selected values of x. If f is twice-differentiable on the interval1 $1 \le x \le 5$, which of the following statements could be true?

先画大概图形，
一阶可导图形是下降的： f'为负
二阶可导图形凹：concave upward ⇒ f' 是 increasing
答案（B）

88. Let f be the function defined by $f(x) = \ln(x^2+1)$. and let g be the function defined by $g(x) = x^5 + x^3$. The line tangent to the graph of f at x=2 is parallel to the line tangent to the graph of g at x equals ax equals a x = a, where ais a positive constant. What is the value of a?

$f(x) = \ln(x^2+1)$

$f'(x) = \dfrac{1}{x^2+1}$

$f'(2) = \dfrac{1}{2^2+1} = \dfrac{1}{5}$

$g(x) = x^5 + x^3$

$g'(x) = 5x^4 + 3x^2 = \dfrac{1}{5}$

$25x^4 + 15x^2 - 1 = 0$

$x^2 = \dfrac{-3 \pm \sqrt{13}}{10} \Rightarrow x = a = \sqrt{\dfrac{\sqrt{13}-3}{10}} = 0.246$

A是0.246， 但是C是0.447?
答案（C??）

89. Let f be a continuous function for all real numbers. Let g be the function defined by $g(x) = \int_1^x f(t)dt$. If the average rate of change of g on the interval $2 \le x \le 5$ is 6, which of the following statements must be true?

$g(x) = \displaystyle\int_1^x f(t)dt \Rightarrow f(x) = g'(x)$

The average value of f on the interval $2 \leq x \leq 5$ is 6
答案（A）

90. For any function f, which of the following statements must be true?

I. If f is defined at $x = a$, then $\lim_{x \to a} f(x) = f(a)$
II. If f is continuous at $x = a$, then $\lim_{x \to a} f(x) = f(a)$
II. If f is differentiable at $x = a$, then $\lim_{x \to a} f(x) = f(a)$

II and III only
答案（C）

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART B ONLY.
DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO

CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of questions—2
A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

1. A particle moves along the *x*-axis so that its velocity at time *t* is given by $v(t)=\frac{t^6-13t^4+12}{10t^3+3}$. At time $t = 0$, the initial position of the particle is $x = 7$.

(a) Find the acceleration of the particle at time $t = 5.1$.

$$a(t)=v'(t)=\frac{(6t^5-52t^3)(10t^3+3)-(30t^2)(t^6-13t^4+12)}{(10t^3+3)^2}$$

$a(5.1)=6.4918$

(b) Find all values of *t* in the interval $0 \le t \le 2$ for which the speed of the particle is 1.

$$v(t)=\frac{t^6-13t^4+12}{10t^3+3}=1$$

$t^6-13t^4+12=10t^3+3$

$t^6-13t^4-10t^3+9=0$

t=0.772 or t=1.401

(c) Find the position of the particle at time $t = 4$. Is the particle moving toward the origin or away from the origin at time $t = 4$? Justify your answer.

$$s(t)=7+\int_0^4 v(t)dt=7+\int_0^4 \frac{t^6-13t^4+12}{10t^3+3}dt=6.712$$

$$v(4)=\frac{4^6-13\times 4^4+12}{10\times 4^3+3}=\frac{780}{643}=1.213>0$$

The particle is moving away from the origin at time t=4.

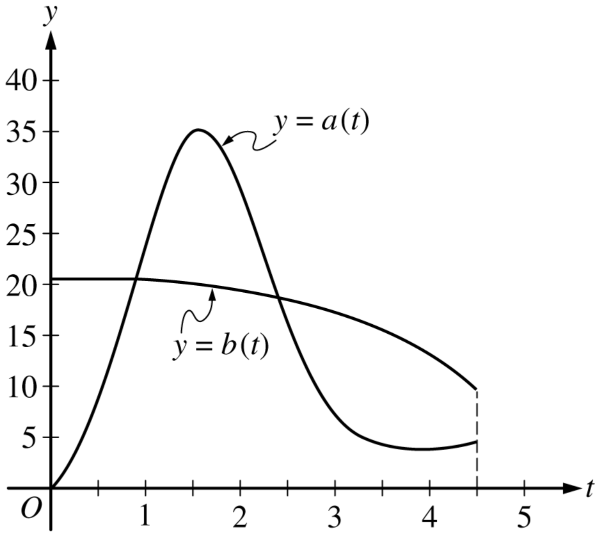
(d) During the time interval $0 < t \le 4$, does the particle return to its initial position? Give a reason for your answer.

$s(0)=7$

$s(1)=9.4>s(0)$

$s(4)=6.7<s(0)$

The particle must have returned to its initial position during the time interval $0<t\le 4$.



2. During the time interval $0 < t \leq 4.5$ hours, water flows into tank *A* at a rate of $a(t) = (2t - 5) + 5e^{2\sin t}$ liters per hour. During the same time interval, water flows into tank *B*at a rate of *b* (t) liters per hour. Both tanks are empty at time *t* = 0. The graphs of *y* = *a(t)* and *y* =*b* (*t*), shown in the figure above, intersect at *t*=*k* and *t*=2.416.

(a) How much water will be in tank *A* at time *t* = 4.5?

$$\int_0^{4.5} a(t)dt = \int_0^{4.5} (2t - 5) + 5e^{2\sin t} dt = 66.53\ (liter)$$

(b) During the time interval $0 \leq t \leq k$ hours, water flows into tank *B* at a constant rate of 20.5 liters per hour. What is the difference between the amount of water in tank *A* and the amount of water in tank *B* at time *t* = *k ?*

Figure a(t) and b(t) intersect at *t*=*k* ⇒ *a(t)=20.5* ⇒ *k=0.892*

$$\int_0^k 20.5dt - \int_0^k a(t)dt = 10.60$$

(c) The area of the region bounded by the graphs of *y*=*a(t)* and *y*=*b(t)* $k \leq t \leq 2.416$ is 14.470. How much water is in tank *B* at time *t* = 2.416 ?

The water in tank B ate time t=2.416 is:

$$\int_0^k 20.5\ dt + \int_0^k b(t)\ dt$$

$$\int_k^{2.416} [(a(t) - b(t)]\ dt = \int_k^{2.416} a(t)\ dt - \int_k^{2.416} b(t)dt = 14.470$$

$$\int_k^{2.416} b(t)dt = \int_k^{2.416} a(t)\ dt - 14.470$$

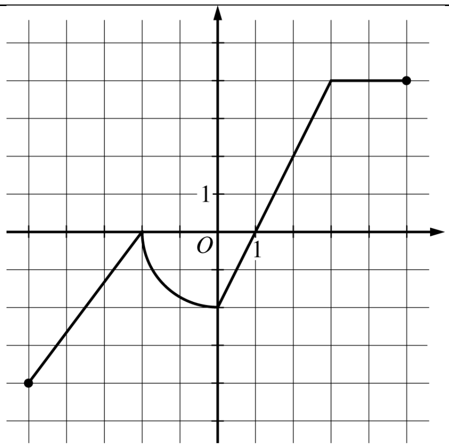
$\int_0^k 20.5\ dt + \int_0^k b(t)\ dt$=48.31

(d) During the time interval $2.7 \le t \le 4.5$ hours, the rate at which water flows into tank *B* is modeled by $w(t) = 21 - \frac{30t}{(t-8)^2}$ liters per hour. Is the difference *w(t)*−*a(t)* increasing or decreasing at time *t* = 3.5 ?
Show the work that leads to your answer.

$w'(3.5) - a'(3.5) = -1.143 < 0$
the difference *w(t)*−*a(t)* is decreasing at t=3.5 .

END OF PART A
IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.
DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB
SECTION II, Part B
Time—1 hour
Number of questions—4
NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of f

3. The graph of the function f, consisting of three line segments and a quarter of a circle, is shown above. Let g be the function defined by $g(x) = \int_1^x g(t)dt$

(a) Find the average rate of change of g from $x = -5$ to $x = 5$.

$$\frac{g(5) - g(-5)}{5 - (-5)} = \frac{12 - (\pi + 7)}{10} = \frac{5 - \pi}{10}$$

(b) Find the instantaneous rate of change of g with respect to x at $x = 3$, or state that it does not exist

$g'(x) = f(x)$
$g'(3) = f(3) = 4$

The instantaneous rate of change of g at x=3 is 4.

(c) On what open intervals, if any, is the graph of g concave up? Justify your answer.

The graph of g is concave up on $-5<x<-2$ and $0<x<3$, because $g'(x)=f(x)$ is increasing on these intervals

(d) Find all x-values in the interval $-5 < x < 5$ at which g has a critical point. Classify each critical point as the location of a local minimum, a local maximum, or neither. Justify your answers.

$g'(x)=f(x)$ is defined at all x with $-5<x<5$

$g'(x)=f(x)=0$ at $x=-2$ and $x=1$.
Therefore, g has critical points at $x=-2$ and $x=1$.

g has neither a local maximum nor a local minimum at $x=-2$ because g' does not change sign there.

g has a local minimum at $x=1$ because g' changes from negative to positive there.

x	0	1	2	3	4	5	6
$f'(x)$	4	3.5	2	0.8	1.7	5.8	7

4. The function f satisfies $f(0) = 20$. The first derivative of f satisfies the inequality $0 \le f'(x) \le 7$ for all x in the closed interval $[0, 6]$. Selected values of f' are shown in the table above. The function f has a continuous second derivative for all real numbers.

(a) Use a midpoint Riemann sum with three subintervals of equal length indicated by the data in the table to approximate the value of $f(6)$.

$$\int_0^6 f'(x)\,dx = 2\cdot 3.5 + 2\cdot 0.8 + 2\cdot 5.8 = 20.2$$

$$f(6) - f(0) = \int_0^6 f'(x)dx$$

$$f(6) = f(0) + \int_0^6 f'(x)dx = 40.2$$

(b) Determine whether the actual value of $f(6)$ could be 70. Explain your reasoning.

$$f'(x) \le 7, \int_0^6 f'(x)dx \le 6\cdot 7 = 42$$

⇒ the actual value of $f(6)$ could not be 70.

(c) Evaluate $\int_2^4 f''(x)dx$

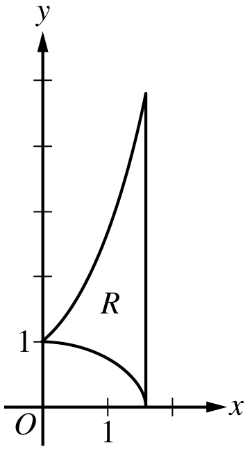
$$\int_2^4 f''(x)dx = f'(4) - f'(2) = 1.7 - 2 = -0.3$$

(d) Find $\lim\limits_{x\to 0} \frac{f(x)-20e^x}{0.5f(x)-10}$.

$$\lim_{x\to 0}(f(x) - 20e^x) = 0$$
$$\lim_{x\to 0}(0.5f(x) - 10) = 0$$

Using L'Hospital's Rule,

$$\lim_{x\to 0}\frac{f(x)-20e^x}{0.5f(x)-10} = \lim_{x\to 0}\frac{f'(x)-20e^x}{0.5f'(x)} = \frac{4-20}{0.5(4)} = -8$$



5. Let R be the region in the first quadrant enclosed by the graph $f(x) = \sqrt{\cos x}$, the graph of $g(x) = e^x$, and the vertical line $x = \pi/2$, as shown in the figure above.

(a) Write, but do not evaluate, an integral expression that gives the area of R.

$$Area = \int_0^{\pi/2} (g(x) - f(x))\, dx$$

(b) Find the volume of the solid generated when R is revolved about the x-axis.

$$Volumn = \pi \int_0^{\pi/2} (g(x)^2 - f(x)^2)\, dx$$
$$= \pi \int_0^{\frac{\pi}{2}} \left((e^x)^2 - \left(\sqrt{\cos x}\right)^2\right) dx$$
$$= \pi \int_0^{\frac{\pi}{2}} (e^{2x} - \cos x) dx$$
$$= \pi \left[\frac{1}{2}e^{2x} - \sin x\right]_{x=0}^{x=\frac{\pi}{2}}$$
$$= \pi \left[\frac{1}{2}e^{\pi} - \sin\frac{\pi}{2} - \left(\frac{1}{2} - 0\right)\right]$$
$$= \pi \left(\frac{1}{2}e^{\pi} - \frac{3}{2}\right)$$

(c) Region R is the base of a solid whose cross sections perpendicular to the x-axis are semicircles with diameters on the xy-plane. Write, but do not evaluate, an integral expression that gives the volume of this solid.

$$Volumn = \frac{1}{2}\int_0^{\frac{\pi}{2}} \pi \left(\frac{g(x) - f(x)}{2}\right) dx$$

6. For $0 \le t \le 6$ seconds, a screen saver on a computer screen shows two circles that start as dots and expand outward.

(a) At the instant that the first circle has a radius of 9 centimeters, the radius is increasing at a rate of 3/2 centimeters per second. Find the rate at which the area of the circle is changing at that instant. Indicate units of measure

$A = \pi r^2$
$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$
$\left.\frac{dA}{dt}\right|_{r=9} = 2\pi \cdot 9 \cdot \frac{3}{2} = 27\pi$

When the radius is 9 centimeters, the area is changing at a rate of $27\pi\ (cm^2/sec)$.

(b) The radius of the first circle is modeled by $w(t) = 12 - 12e^{-0.5t}$ for $= 0 \le t \le 6$, where $w(t)$ is measured in centimeters and *t* is measured in seconds. At what time *t* is the radius of the circle increasing at a rate of 3 centimeters per second?

$w'(t) = (-12)(-0.5)e^{-0.5t} = 6e^{-0.5t}$

$6e^{-0.5t} = 3$
$\Rightarrow e^{-0.5t} = \frac{1}{2}$
$\Rightarrow -0.5t = \ln\frac{1}{2}$
$\Rightarrow t = 2\ln 2$

The radius is increasing at a rate of 3 centimeters per second at time $t = 2\ln 2$ seconds.

(c) A model for the radius of the second circle is given by the function *f* for $0 \le t \le 6$, where *f(t)* is measured in centimeters and *t* is measured in seconds. The rate of change of the radius of the second circle is given by $f'(t) = t^2 - 4t + 4$. Based on this model, by how many centimeters does the radius of the second circle increase from time *t*=0 to *t*=3 ?

$\int_0^3 (t^2 - 4t + 4)dt = \left[\frac{1}{3}t^3 - 2t^2 + 4t\right]_{t=0}^{t=3}$
$= \left(\frac{1}{3}\cdot 3^3 - 2\cdot 3^2 + 4.3\right) - \left(\frac{1}{3}\cdot 0^3 - 2\cdot 0^2 + 4.0\right)$
$= 9 - 18 + 12 = 3$

The radius increases by 3 centimeters from time *t*= 0 to time *t=3* seconds.

STOP
END OF EXAM